



# The design optimization of a small axial turbine with millions of configurations

The case for computerized optimization over  
manual design interventions



## Introduction

Increasing worldwide interest in expendable, low-cost turbojet engines for both civil and military purposes has led to the design and production of many miniature turbojet engines by companies and groups of enthusiasts in Germany, the UK, Spain, Taiwan, Denmark, Australia, the Netherlands, the USA, Russia, Japan, Israel and other countries [1]. These engines are usually equipped with a small low-pressure turbocharged centrifugal compressor driven by a low-pressure ratio, one-stage, axial turbine. Their engine performance is also quite low, with the specific fuel consumption (SFC) being in the range of 1.3-2.0 kilograms of fuel per kilogram of force (kg/kgf).

The large relative aerodynamic losses due to the small size of the components are responsible for the low performance of these miniature turbojet engines. On the other hand, the main components of these engines are usually not optimized because of the low cost of the engines and of their design.

This article describes a simple method to optimize the performance of a small axial turbine during the preliminary design stage by using a fast mean-line design code with the modeFRONTIER optimization software and a well-established genetic algorithm

## Mean-line axial-flow turbine analysis

The layout of a miniature turbojet engine is shown schematically in Figure 1. Its main elements are a centrifugal compressor, a combustion chamber, an axial power turbine, and a converging propelling nozzle. A compressor increases the air pressure and pushes the air towards the combustion chamber, where the air temperature raises due to fuel

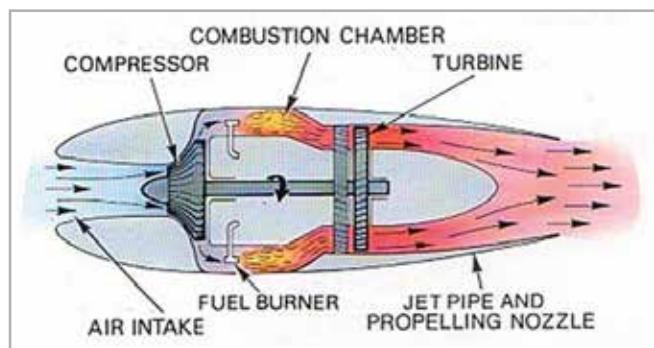


Fig. 1 - Miniature turbojet engine layout

combustion. These hot gases in the turbine produce the power to drive the compressor. They are then expelled through the propelling nozzle to produce the engine thrust.

The main objectives of a turbine's design are to convert the kinetic energy of the gas into the shaft power with maximum efficiency and to provide an axial non-swirling exit of the gas into the propelling nozzle. Additional objectives are withstanding the mechanical and thermal stresses, a necessary safety margin from resonance conditions, manufacturability, smooth blade profiles,

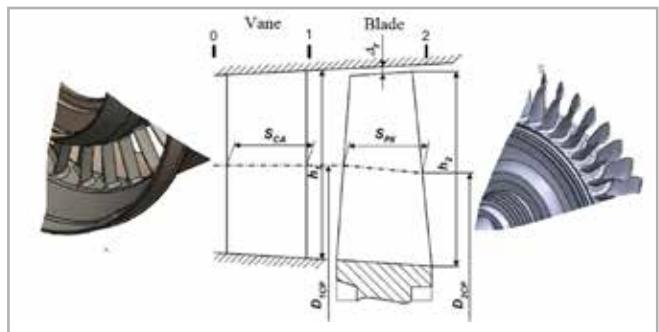


Fig. 2 - Axial turbine schematic and construction

etc. The power turbine in a miniature turbojet engine is usually a one-stage axial turbine, shown schematically in Figure 2. These turbines consist of static vanes that direct the flow into the rotating blades at the correct angle to provide momentum to the shaft. For

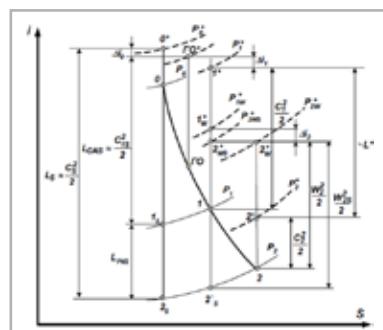


Fig. 3 - Expansion through the turbine stage

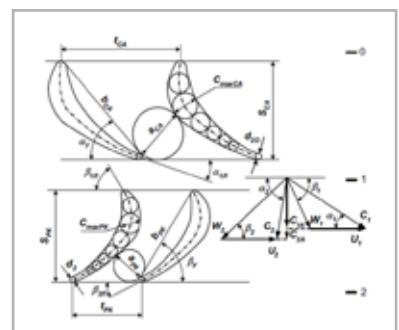


Fig. 4 - Vane and blade schematic

maximum turbine efficiency, the vanes and the blades must be perfectly aerodynamically matched. This is achieved by means of the careful design of the geometry of the vanes and the blades, known as mean-line analysis.

Turbine design is a multi-stage process where the first stage includes the analysis of mean-line performance, the setting of the turbine dimensions, the definition of the blade inlet-outlet angle, the selection of an aerodynamic loss model, and the efficiency calculation. The design of the vanes and blades is also subject to additional constraints, such as the profile smoothness. To obtain a high-quality preliminary turbine design, we used a two-dimensional calculation of the turbine parameters at five sections along the blade, instead of the usual one-dimensional, mean-line analysis at the median radius. Since a large number of input parameters affect the turbine's performance and the flow's exit angle, we used a multi-objective, multi-constraint and multi-variable optimization to improve the turbine's performance at this stage.

Axial turbine thermodynamic analysis is based on an enthalpy-entropy diagram of the process of gas expansion in the turbine stage, as shown in Figure 3. The relationships between the turbine parameters are determined by the geometry of the vanes and blades and of the velocity triangles, as presented in Figure 4. We varied these velocity triangles with each blade/vane section, while a profile stacking around a center of mass, shown in Figure

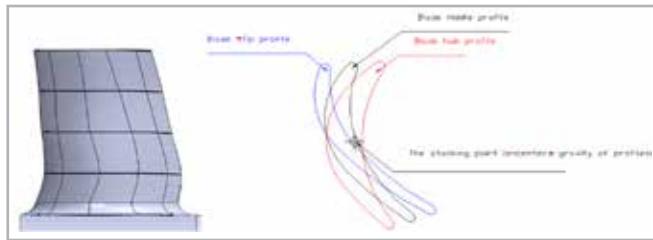


Fig. 5 - Blade sections and profiles stacking

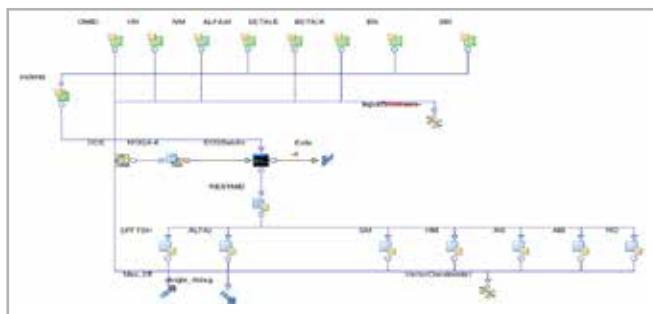


Fig. 6 - Optimization model

5, allowed us to reduce the blade/vane stresses and to achieve better turbine performance. Profile stacking produces a quasi-three-dimensional form in the vanes and blades, which generates a significant improvement compared to the one-dimensional design of the median radius.

The fast Fortran-compiled, in-house code for two-dimensional mean-line analysis of an axial-flow turbine written by the late R. Priamovsky was used for this project. The program input file includes a large number of parameters, such as turbine mass flow rate, stage inlet temperature and pressure, obtained at the outlet of the combustion chamber, outlet pressure, pressure recovery in the inlet duct, gas properties, rotational speed, the hub and tip radii of the vanes and blades, their numbers, radial clearance, hub leakage temperature, inlet angles, outlet velocity coefficients, preliminary estimates of velocity loss coefficients, and blade profile characteristics. In addition, the vane and blade inlet and outlet angles, and trailing edge thicknesses are provided for each of the five radial sections. All this data is provided in the constant textual tabular form that allows the optimization model to conveniently change all the input parameters.

Mean-line turbine analysis uses the appropriate losses model and conventional blade/vane shapes, and provides the user with the radial distribution of the velocity triangles, pressures, temperatures, densities, etc. Turbine power, friction losses, stress at the hub and efficiency are also calculated and written into the output ASCII file.

A reliable model of the energy losses in the flow path of an axial-flow turbine allows the accurate prediction of the turbine's characteristics in 1D and 2D mean-line analysis. As mentioned in [2], more than ten complex models are known today, together with dozens of equations calculating the different loss components. These models and loss equations have been obtained over the last 70 years by researchers around the world. The authors of [2] analyzed the results of experimental investigations of the profile

losses for more than 170 non-swirling cascades of axial turbines with a constant height section. These experiments were performed by several gas turbine manufacturers from the former USSR for many axial-flow turbine blade profiles used in aircraft gas turbine engines. Based on this data, the accuracy of the five most popular loss models was compared and summarized in [3]. As was shown, the CIAM model of axial-flow turbine losses is the best of the five popular models for a wide range of turbines from the perspective of maximum error and standard deviation. Therefore, we used this model for the present analysis.

### Optimization module

The code for a 2D axial turbine mean-line analysis is written in Fortran IV and compiled into an executable (exe) file. This code includes 19 subroutines including the CIAM empirical loss model and is able to treat both subsonic and supersonic flow conditions in the turbine. After initiation, the executable file reads the input file, performs the calculations and writes the results into a formatted output file in ASCII format.

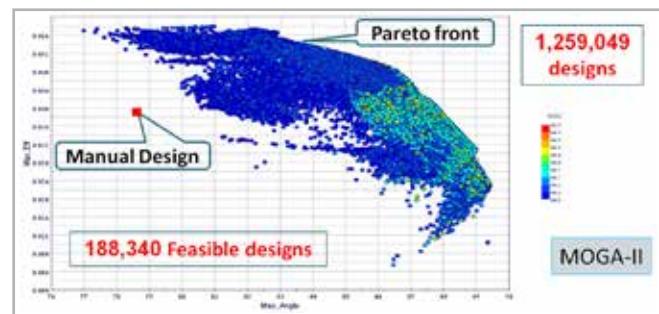


Fig. 7 - Optimization with  $N > 1,000,000$  designs

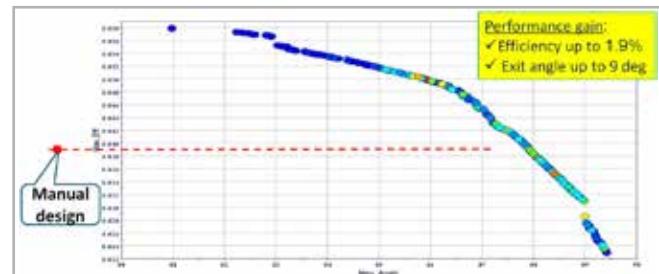


Fig. 8 - Pareto designs (first optimization)

The optimization of the turbine's performance was undertaken using the modeFRONTIER (mF) tool, developed by ESTECO. A general layout of the optimization model is shown in Figure 6. The mF workspace consists of five main blocks: the input parameters block, including the constraints; the design of experiments (DoE) block, where the initial (parent) population of parameters is randomly chosen; the optimizer block, where the initial parameters are changed; the execution block that produces the output file; the target block with the constraints on the output parameters. The input and output files are connected by the user-defined node that allows the user to perform command-line operations on the executable file.

In the present study, the multi-objective genetic algorithm (MOGA-II) was used to treat a large number of input parameters

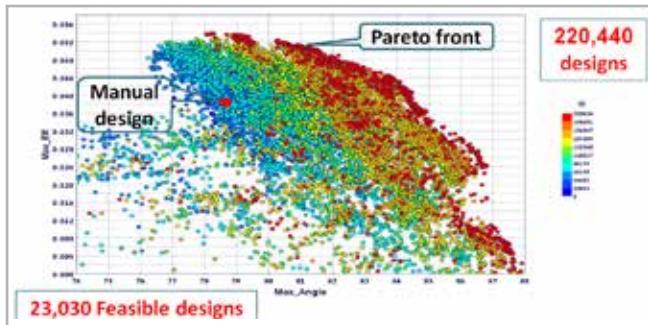


Fig. 9 - Optimization with limited blade angle of attack

and the numerous constraints on the output parameters. Two design objectives were defined: the turbine's thermodynamic efficiency, and the mean flow angle at the turbine's outlet. A minimal swirl of the outlet flow allows maximum thrust to be developed in the turbojet engine's propelling nozzle. Our model included 33 input variables and 20 output parameters, including turbine efficiency, shaft power and the blade stress estimation. The model under consideration works under the limitations of 139 constraints including 66 min/max constraints on the input variables, 38 constraints on the geometrical interactions, 15 constraints on the monotony of the radial vane/blade profiles and stacking, and the 2 design objectives namely the requirements for the mass flow rate and the turbine power. Among other limitations was the requirement for the number of vanes and blades to be a prime number to reduce the likelihood of harmful resonances. It should be noted that, at the preliminary stage of the manual analysis by a very experienced designer, this constraint prevented the attainment of an acceptable level of efficiency and was omitted. The limitation was returned and fulfilled by the computerized optimization process. Based on the large number of initial variables, and to obtain a representative variability in the initial (parent) population, 10,001 DoE sets of variables were formed using the Sobol deterministic DoE algorithm. The Sobol algorithm mimics a random choice to obtain a uniform sampling of the design space and to reduce the clustering of parameters. The output parameters were also limited by the defined power of the turbine and the degree of reaction at each section. A total of 18 constraints limited the output.

Because of the large dimension of the problem and the complexity of the constraints, the optimization was performed using the MOGA-II genetic algorithm. The elitism procedure of this algorithm, which preserves excellence, ensures that each new generation's performance is greater than the performance of its parent generation.

## Results

At the first attempt, the optimization model was run through 126 generations to produce 1,259,049 designs. Because of the strict constraints on the output parameters, only 188,340 of the designs (about 15%) were feasible. Figure 7 clearly demonstrates the superiority of the computerized optimization over the manual design. Figure 8 shows the Pareto frontier of the best designs from

the first optimization, which generated a 1.9% performance gain in efficiency and 9 degrees of the exit angle. The Pareto graph shows a few discontinuities that clearly demonstrate the non-linear nature of this problem and the advantage of using a genetic algorithm in this case. Despite the success of the first phase of the turbine design, a more rigorous analysis of the optimized designs presented in Figure 8 revealed unexpectedly large angles of attack ( $DBeta_1$ ) between the blades and the gas flow (up to 20 degrees). The problem, here, is that the turbine loss model has not been validated at such angles, so another limiting condition was added in the form  $|DBeta_1| \leq 10\text{deg}$ . A further 220,440 configurations were calculated using the Non-dominated sorting genetic algorithm II (NSGA-II). According to mF [4], NSGA-II implements the crowding distance approach which guarantees the diversity and spread of the solutions on the Pareto front by estimating the

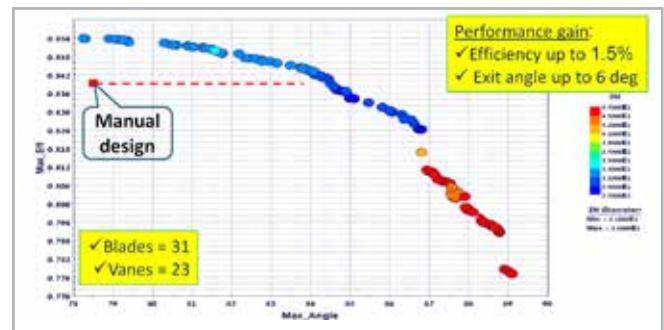


Fig. 10 - Pareto designs (second optimization)

density of the solutions in the objective space and guiding the selection process towards a uniform spread. The points belonging to the same front are sorted such that a higher ranking is given to the points located in the less populated regions of the front. Since this sorting demands additional calculations, we used the NSGA-II algorithm because of the smaller population. No advantage over the MOGA-II algorithm was observed.

After the addition of a constraint on the blade's angle of attack, only 23,030 feasible designs were obtained (about 10%) from this second optimization. Figure 9 shows that the best efficiency reached was 85.4%, which reduced to 85.2% when the exit angle was above 80 degrees (generally, the exit angle should be close to 90 degrees).

Figure 10 shows the Pareto frontier of the best designs from the second optimization. A prime number of vanes (23) and blades (31) was obtained, with a potential performance gain of up to 1.5% in efficiency and 6 degrees of the exit angle.

To maximize the efficiency of the optimized turbine, another 546,000 designs were calculated under the following conditions:

- Vane number ZN=23 (primary number);
- Blade number ZM=29 & 31 (primary numbers);
- Exit angle limited by 80 degrees;
- Maximum efficiency was the only target.

These additional conditions reduced the number of design variables and objectives, making it easier to find a solution.

## CASE STUDIES

As a result, the turbine's efficiency was improved up to 85.5% (see Figure 11). As one can see, the main gain in the turbine's efficiency was already achieved after 20-30,000 designs, while a very moderate, but clear improvement was obtained after that. As already mentioned, 33 input variables were used in the design of the small axial-flow turbine. Obviously, not all these parameters equally affecting the turbine's performance. A sensitivity analysis was performed using the mF tool [4] to detect the most important input variables. This enabled the exclusion of certain variables

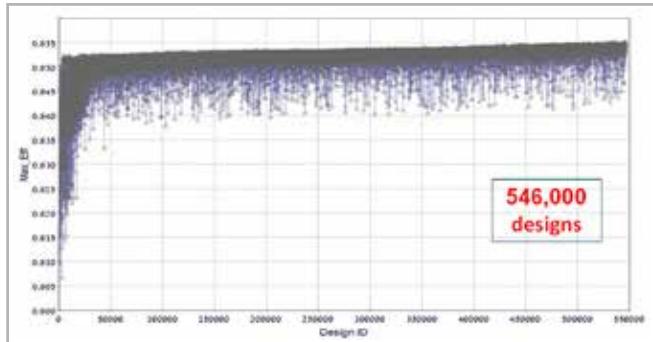


Fig. 11 - Efficiency-only optimization

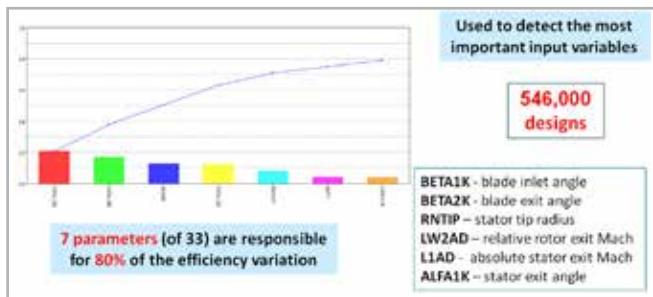


Fig. 12 - Sensitivity analysis

from the optimization, reducing the required computational effort. This analysis is particularly useful to better understand the physical model. In particular, sensitivity analysis functions can be used in the Response surface method (RSM) training process.

Figure 12 illustrates some of the sensitivity analysis results. The following 7 of the 33 parameters were responsible for 80% of the efficiency variation:

- BETA1K – blade inlet angle
- BETA2K – blade exit angle
- RNTIP – stator tip radius
- LW2AD – relative rotor exits Mach number
- L1AD – absolute stator exit Mach number
- ALFA1K – stator exit angle

The second section from the turbine disk, next to the hub, was found to be the most important section for maximum efficiency.

It is to be noted that the huge amount of data generated through the optimization process becomes difficult to treat: we found that the mF operations in the working table became slow at about 500,000 designs. Large tables cannot be treated by MS Excel due to its limit of 1,048,576 rows and 32,000 graph points. The optimization

process was therefore split to cater for these limitations, and only the feasible designs were treated in post-processing analysis.

As a result of the optimization, the turbine exit angle was improved by 2–6 degrees, while the efficiency was increased by 1.5%. One may question how meaningful this result is. Visual inspection of Figure 7 demonstrates a sufficient improvement in optimized efficiency compared to the manual design that was based on a few dozen attempts by a turbine design expert. However, some quantitative estimation of the performance improvement gained through the optimization is desirable. To this end, we used the classical Smith chart [5], which is a map that describes the empirical correlations between the efficiency of the state-of-the-art axial turbine stages, and the loading and flow coefficients, and which is widely accepted as feasible for use during preliminary turbine design. According to the Smith chart, the turbine efficiency achieved with the manual design was about 4% below the maximum efficiency of the axial turbine under consideration. This meant that the efficiency improvement of 1.5% achieved with design optimization had closed about 40% of the potential gain in turbine efficiency. Other measures, such as the advanced blade geometries, hub and tip contouring, abradable seals, stacking, and 3D blade design can close the remaining gap.

## Conclusion

Fast computer code and the advanced modeFRONTIER optimization and analysis tools allowed us to perform an optimal mean-line design of a small axial turbine with more than 2,000,000 configurations. The time that was spent on this project, about 80 work hours, is usually sufficient to manually check a few dozen designs only. 33 parameters and 139 constraints were taken into account. The turbine exit angle was improved by 2–6 degrees, and its efficiency was increased by 1.5%, closing about 40% of the potential efficiency gain for this type of turbine. We recommend using mF models with no more than 500,000 designs simultaneously.

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